reviewer has examined, particularly because of its format, which is to be contrasted with the customary semiquadrantal arrangement. The tabulation of the versine and doversine clearly requires the extended range of the argument given in these tables. The author justifies the tabulation of the six standard trigonometric functions also over the first two quadrants on the basis of the resulting ease of application to the solution of triangles. The typography is generally excellent, and the arrangement of the tabular data convenient. This book should prove a useful addition to the literature of mathematical tables.

J. W. W.

30 [F].—R. Kortum & G. McNiel, *A Table of Periodic Continued Fractions*, Lockheed Aircraft Corporation, Sunnyvale, California, 1961, xv + 1484 p., 29 cm.

This huge and interesting table contains, first, the *half-period* of the regular continued fraction for the $\sqrt{D}$ for each non-square natural number D less than 10,000. For example, since

$$\sqrt{13} = 3 + \frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{3+\sqrt{13}},$$

under $D = 13$ are listed the *partial quotients*: 3, 1, 1. Again, since

$$\sqrt{19} = 4 + \frac{1}{2+}\frac{1}{1+}\frac{1}{3+}\frac{1}{1+}\frac{1}{2+}\frac{1}{4+\sqrt{19}},$$

under $D = 19$ is the list: 4, 2, 1, 3.

Next, let

$$\sqrt{D} = q_0 + \frac{1}{q_1+}\frac{1}{q_2+}\cdots \quad \text{and} \quad x_i = q_i + \frac{1}{q_{i+1}+}\cdots$$

and $x_i = (\sqrt{D} + P_i)/Q_i$. Then Q_i, the *denominators of the complete quotients*, are listed in a row parallel to the q_i.

If p, the *period* of the continued fraction, is odd, as in $p = 5$ for $D = 13$, the table gives the smallest solution x, y of

$$x^2 - Dy^2 = -1.$$

If p is even, as in $p = 6$ for $D = 19$, the smallest solution is given of the so-called Pell equation:

$$x^2 - Dy^2 = +1.$$

The values of D for which p is odd are marked with an asterisk.

Finally, if $p^2/D > 1$, this ratio is given to 9 decimals. All of this was computed on an IBM 7090 in 36 minutes.

While in [1] the continued fractions for $D < 10{,}011$ have already been given, together with *both* sequences P_i and Q_i defined above, the range here of D for the solutions x, y, and for the ratio p^2/D, would appear to exceed that in any published table.

From a theoretical point of view the quantity p^2/D is of considerable interest. If we list those D where p^2/D attains a new maximum we obtain the following table:

D	p^2/D	D	p^2/D
3	1.333	631	3.651
7	2.286	919	3.917
43	2.326	1726	4.487
46	3.130	4846	4.768
211	3.204	7906	4.948
331	3.492		

Further, the authors have extended their computation to $D = 51{,}000$ and have listed all such D for which p^2/D exceeds 5. We may thus continue:

D	p^2/D	D	p^2/D
10651	5.141	19231	5.731
10774	5.257	32971	5.819
18379	5.641	48799	6.064

While these empirical data obviously suggest the possibility that

$$p = O(D \log D)^{1/2},$$

the authors refrain from such a suggestion and also from any reference to pertinent theoretical work.

Tenner's algorithm was used in the computation of the continued fractions. This requires two divisions and subtractions and one multiplication and addition in each cycle. An alternative algorithm is known that replaces one of these divisions by an addition. This latter computation would therefore be somewhat faster. But, alternatively, the redundancy in Tenner's algorithm (implicit in the fact that in one of these divisions the remainder is always zero) allows for a check at each cycle. But whether the authors utilized this check, or indeed made any check, is not indicated in their introduction.

There is a printing defect which could have been easily avoided. In a block of 10 decimal digits, if the high-order digit is a zero, it is printed as a blank. Thus, for $D = 801$, $x = 500002000001$ and $y = 17666702000$ are printed as

```
50     2000001
 1  7666702000
```

This is because the binary to decimal integer conversion subroutine which was used *deliberately* causes such suppression of high-order zeros on the assumption that they will not be preceded by a significant digit. To circumvent such suppression one can add 10^{10} to each binary number before conversion to 10 decimal digits. This *fictitious* high-order 1 is an 11th digit, and will not be printed, since only the 10 low-order digits will be converted. Nonetheless, the routine is deceived, by its presence, into thinking that the high-order zeros are not now high order. A programmer who keeps on his toes can often outwit the makers of subroutines.

In a covering letter one of the authors indicates that this table is the second [2] in a series of fourteen, or more, number-theoretic tables. While a few of these duplicate, at least in part, some known tables, the latter are often on magnetic tape, or cost money, or are otherwise inaccessible. The entire proposed series will certainly be welcome to mathematicians working in number theory.

D. S.

1. WILHELM PATZ, *Tafel der regelmässigen Kettenbrüche*, Berlin Akademie-Verlag, 1955.
2. The first is *A Table of Quadratic Residues for all Primes less than 2350*. See RMT 35, *Math. Comp.*, v. 15, 1961, p. 200.

31 [I].—HERBERT E. SALZER & CHARLES H. RICHARDS, *Tables for Non-linear Interpolation*, 11 + 500 p., 29 cm., 1961. Deposited in the UMT file.

These extensive unpublished tables present to eight decimal places the values of the functions $A(x) = x(1 - x)/2$ and $B(x) = x(1 - x)(2 - x)/6$, corresponding to $x = 0(10^{-5})1$. This subinterval of the argument is ten times smaller than that occurring in any previous table of these functions.

These tables can be used for either direct or inverse interpolation, employing either advancing or central differences. In the introductory text are listed, with appropriate error bounds, the Gregory-Newton formula and Everett's formula, for direct quadratic and cubic interpolation, and formulas for both quadratic and cubic inverse interpolation, employing advancing differences and central differences. Examples of the use of these formulas are included.

The convenience of these tables is enhanced by their compact arrangement, which is achieved by tabulating $B(1 - x)$ next to $B(x)$. This juxtaposition, in conjunction with the relation $A(1 - x) = A(x)$, permits the argument x to range from 0 to 0.50000 on the left of the tables, while the complement $1 - x$ is shown on the right.

The authors note the identity $A(x) - B(x) \equiv B(1 - x)$, which can be used as a check on interpolated values of $A(x)$, $B(x)$ and $B(1 - x)$, and also as a method of obviating interpolation for $B(1 - x)$, following interpolation for $A(x)$ and $B(x)$.

Criteria for the need of these interpolation tables are stated explicitly, with reference to both advancing and central differences.

A valuable list of references to tables treating higher-order interpolation is included.

The authors add a precautionary note that this table is a preliminary print-out, not yet fully checked.

J. W. W.

32 [I, X].—GEORGE E. FORSYTHE & WOLFGANG R. WASOW, *Finite-Difference Methods for Partial Differential Equations*, John Wiley & Sons, Inc., New York, 1960, x + 444 p., 23 cm. Price $11.50

The solution of partial differential equations by finite-difference methods constitutes one of the key areas in numerical analysis which have undergone rapid progress during the last decade. These advances have been accelerated largely by the availability of high-speed calculators. As a result, the numerical solution of many types of partial differential equations has been made feasible. This is a development of major significance in applied mathematics.